## AP Statistics

Module 7 Free Response and Essay Tips

Below you will find a breakdown of different AP topics for this module. The sections include expectations for answering questions over each topic and examples of how these areas should be handled:

## BASICS FOR T TESTS AND T CONFIDENCE INTERVALS

| Actual AP Exam Expectations | Notes |
| :---: | :---: |
| T test and interval conditions | State if: Population standard deviation is unknown, SRS, normality, independence, Pop > 10n <br> Create a boxplot and state the shape and if any outliers <br> *If $n<15$ the data must give an approx. normal distribution with little skewness and outliers. PROCEED WITH EXTREME CAUTION! <br> *If $15<n<40$ the data must give a distribution that does not have extreme outliers or extreme skewness. PROCEED WITH CAUTION!! <br> *If $n>40$, it is justified to use a t procedure because the CLT applies |

CONFIDENCE INTERVALS -- 1 sample and 2 sample and MARGIN OF ERROR

| Actual AP Exam Expectations | Notes |
| :---: | :---: |
| 1. SHOW all steps of the confidence interval | 1) Parameter: $\mathbf{1}$ sample tinterval "We want to estimate the mean, $\mu$, of context of problem " <br> Matched pairs ( 1 sample $t$ interval) "We want to estimate the mean difference of $\qquad$ context between sample 1 and sample 2 in context. <br> 2 sample tinterval "We want to estimate the mean difference between population 1 and population 2 in context of problem" <br> 2) Conditions ** For 2 sample T interval, you must check and state conditions for both populations** <br> *Population standard deviation is unknown <br> *SRS: If stated in problem, tell the grader, if not, then you should say: We are not told if SRS of all $\qquad$ , so proceed with caution! <br> *Normality $-\mathrm{n}<15, \quad 15<\mathrm{n}<40, \quad \mathrm{n}>40$ <br> SEE \#3 ABOVE in condition section <br> *Independence - Each observation is independent and population > 10 n ie: if I have 100 calculators, we tell AP Grader, we can assume that each calculator is independent and there are more than 10(100) or 1000 calculators <br> Name the interval: One sample $t$ interval or 2 sample $t$ interval |


|  | 3) Show all work: <br> If you use you the formula, you must do all work by hand and show the equation. If you use the calculator you must NAME the test/interval that you selected, ALL inputted information, and ALL output information If DATA is given, graph and describe <br> 4) Interpret the results in the context of the problem and make a connection to the given information. Remember the 3 C's: Context, connections and conclusion. |
| :---: | :---: |
| 2. Degrees of Freedom | 1 sample t interval: Sample size -1 , which is $\mathrm{n}-1$ <br> 2 sample tinterval: find the degrees of freedom on your calculator when you compute the interval |
| 3. Interpret the results of the confidence interval | Conclusions should be given in terms of the context of the question. 1 sample tinterval: We are $\qquad$ \% confident that the true population mean $\mu$ of $\qquad$ context will be between lower value and upper value <br> Matched pairs (1 sample $t$ interval): We are $\qquad$ \% confident that the mean of the population differences of $\qquad$ context is between lower value and upper value <br> $\mathbf{2}$ sample tinterval: We are $\qquad$ \% confident that the true difference between the population mean $\mu_{1}$ of $\qquad$ context and the population mean $\mu_{2}$ of $\qquad$ is between lower value and upper value |
| 4. T interval equations | 1 sample Tinterval: $\bar{x} \pm$ critical value $\left(\frac{s}{\sqrt{n}}\right)$ <br> Matched Pairs (1 sample T interval): $\overline{X_{d}} \pm \frac{s_{d}}{\sqrt{n}}$ <br> *use the mean and standard deviation of the differences* <br> 2 sample T interval: $\left(\bar{x}_{1}-\bar{x}_{2}\right) \pm t^{*} \sqrt{\frac{s_{1}^{2}}{n_{1}}+\frac{s_{2}^{2}}{n_{2}}}$ |
| 5. Margin of error | Everything after the $+/-$ in the confidence interval This value shows how accurate we believe our guess is and is based on the variability of the estimate |
| 6. Find the $t^{*}$ for the confidence interval | Using the calculator: Subtract the level of significance from 1. So 1- $\alpha$ and then divide that value by 2 . On the calculator DISTR-> INV T -> AREA: ((1- $\alpha) / 2)$ df: $\mathrm{n}-1->$ ENTER This will give you the critical value t* |

- The Margin of error decreases when: $\mathrm{t}^{*}$ decreases, confidence level decreases, sample size ( n ) increases
- Increasing the sample size decreases the width of a confidence interval and the variability
- Use $t$ when you are not given the population standard deviation.
- Use a one sample t interval when you have matched pairs - one sample that is being analyzed at two different times, and we are analyzing the actual difference amount.
- Use a two sample t interval when you have two different samples that are being analyzed and compared to one another to see if they are different.
- When 0 is in the confidence interval, we can assume that there is no difference between the 2 means.
- Here is a video specific to this topic. It includes examples and how to be most successful on the AP exam for the topic.
7.01: Standard Error and T Intervals
https://sas.elluminate.com/site/external/jwsdetect/playback.jnlp?psid=2015-01-
24.0743.M.02B50E368656D296A2DCBFED1F5B9E.vcr\&sid=679


## TESTS OF SIGNIFICANCE - One sample T test and 2 sample T test

| 1. SHOW all steps of the test | 1) Parameter: same as confidence intervals <br> 2) Conditions ** For 2 sample $T$ test, you must check and state conditions for both populations** (see above interval \#1 for specifics on each of these) <br> * Population standard deviation is unknown <br> *SRS <br> *Normality <br> *Independence <br> Name the test: One sample $t$ test or 2 sample $t$ test <br> State the null ( Ho ) and alternative hypotheses ( Ha ) in context. <br> Note: $\mathrm{H}_{0}$ : null hypothesis is ALWAYS = to <br> $\mathrm{H}_{\mathrm{a}}$ : alternative is < > or $\neq$ <br> 3) Show all work: same as for confidence intervals <br> 4) Interpret the results in the context of the problem and make a connection to the given information. Remember the $\mathbf{3 C \prime s}$ |
| :---: | :---: |
|  |  |


| 2. State hypotheses in context | 1 sample t test: <br> Ho: The true population mean of $\qquad$ context is equal to $\qquad$ . $\mu=\mu_{0} *$ You must at least define what $\mu$ is* <br> Ha: The true population mean of $\qquad$ context is $<>\neq$ $\qquad$ . $\mu<>\neq \mu_{0}$ <br> Matched pairs t test: <br> Ho: The mean of the differences between __context population 1 and context population 2_is 0 . $\mu_{1}-\mu_{2}=0 \quad \text { or } \quad \mu_{1}=\mu_{2}$ <br> *You must define what $\mu_{1}$ and $\mu_{2}$ are* <br> Ha: The mean of the of the differences between __context population 1 and context population 2_ is s $<>\neq 0$. $\mu_{1}-\mu_{2}<>\neq 0 \quad \text { or } \quad \mu_{1}<>\neq \mu_{2}$ <br> 2 sample t test: <br> Ho: The mean context population 1 is equal to the mean context population 2_. $\mu_{1}-\mu_{2}=0 \quad \text { or } \quad \mu_{1}=\mu_{2}$ <br> *You must define what $\mu_{1}$ and $\mu_{2}$ are* <br> Ha: The mean context population 1 is $<>\neq$ to the mean context population 2. $\mu_{1}-\mu_{2}<>\neq 0 \quad \text { or } \quad \mu_{1}<>\neq \mu_{2}$ |
| :---: | :---: |
| 3. T test equation ( 1 sample) | 1 sample T test: $\frac{\bar{x}-\mu}{\frac{\mathrm{s}}{\sqrt{\mathrm{n}}}}$ <br> Matched Pairs (1 sample T test): $\frac{\overline{x_{d}}-\mu_{0}}{\frac{s_{d}}{\sqrt{n}}}$ <br> *use the mean and standard deviation of the differences* |
| 4. T test equation ( 2 sample) | $2 \text { sample T test: } \frac{\left(\bar{x}_{1}-\bar{x}_{2}\right)}{\sqrt{\frac{s_{1}^{2}}{n_{1}}+\frac{s_{2}^{2}}{n_{2}}}}$ |
| 5. Interpret the results of a t test | Conclusions should be given in terms of the context of the question. <br> 1 sample t test: Reject or Fail to reject the null hypothesis that $\qquad$ context of the problem because the $p$-value is < > $\neq$ level of significance. There is/is not sufficient evidence to suggest that $\qquad$ restate the null hypothesis |


| Interpret the results of a t test | Matched pairs $t$ test: Reject or Fail to reject the null hypothesis that <br> the mean differences of _sample 1 and sample 2 is equal to 0 because <br> the $p$ value is $><\neq$ level of significance. There is/is not sufficient <br> evidence to suggest that__restate the null hypothesis |
| :--- | :--- |
| 2 sample $t$ test: Reject or Fail to reject the null hypothesis that the |  |
| difference in the population mean of population $1 \_$and the |  |
| population mean of_population $2 \_$is equal to 0 because the $p$ value |  |
| is $><\neq$ level of significance. There is/is not sufficient evidence to |  |
| suggest that_restate the null hypothesis |  |.

- P-value: The probability of seeing a result from a random sample that is as extreme as or more extreme than the result you got from your random sample, if the null hypothesis is true.
- You can also find the $p$ value, once you have the $t$ score, by using $2^{\text {nd }}->$ Distr $->$ tcdf(lower, upper, df)
- Increasing the sample size decreases the $p$-value of the test (making the rejection of the null more convincing). As n increases, so does the power of the test.
- Use a two sample $t$ test when you have two different samples that are being analyzed and compared to one another.
- Use a one sample tinterval when you have matched pairs - one sample that is being analyzed at two different times, and we are analyzing the actual difference amount.
- Here is a video specific to this topic. It includes examples and how to be most successful on the AP exam for the topic
7.02-7.06: Significance tests and Difference of Means
https://sas.elluminate.com/site/external/jwsdetect/playback.jnlp?psid=2015-01-
24.0954.M.02B50E368656D296A2DCBFED1F5B9E.vcr\&sid=679


## CONFIDENCE INTERVALS -- 1 and 2 proportions and MARGIN OF ERROR

| Actual AP Exam Expectations | Notes |
| :---: | :---: |
| 1. SHOW all steps of the confidence interval | 1) Parameter: "We want to estimate the mean proportion, $p$, of context of problem " <br> 2) Conditions ** For 2 proportion intervals, you must check and state conditions for both populations** <br> *SRS: If stated in problem, tell the grader, if not, then you should say: We are not told if SRS of all $\qquad$ , so proceed with caution! <br> *Normality $-n p>5$ and $n(1-p)>5$ If these are satisfied then we must say that we assume normality. <br> *Independence - Each observation is independent and population > 10n <br> Name the interval: One proportion or 2 proportion interval <br> 3) Show all work: same as tintervals <br> 4) Interpret the results in the context of the problem and make a connection to the given information. Remember the $\mathbf{3 C \prime s}$ : Context, connections and conclusion. |
| 2. Interpret the results of the confidence interval | Conclusions should be given in terms of the context of the question. 1 proportion interval: We are $\qquad$ \% confident that the true population proportion of $\qquad$ context will be between lower value and upper value <br> 2 proportion interval: We are $\qquad$ \% confident that the true difference between $\qquad$ and $\qquad$ context is between lower value and upper value . |
| 3. 1 proportion interval | $\widehat{p} \pm z^{*} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ |
| 4. 2 proportion interval | $\hat{p}_{1}-\hat{p}_{2} \pm z^{*} \sqrt{\frac{\hat{p}_{1}\left(1-\hat{p}_{1}\right)}{n_{1}}+\frac{\hat{p}_{2}\left(1-\hat{p}_{2}\right)}{n_{2}}}$ |
| 5. Margin of error | Everything after the $+/$ - in the confidence interval <br> This value shows how accurate we believe our guess is and is based on the variability of the estimate |
| 6. Equation to find sample size | $z^{*} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq m$ *You will be given $m$ (margin of error) |

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- When 0 is in the confidence interval, we can assume that there is no difference between the 2 proportions.
- If $p$ is or $\hat{p}$ is not stated, we assume it is 0.5
- When estimating sample size, round up ALWAYS!
- $\hat{p}=\frac{\text { total number of successes in the sample }}{\text { total number of individuals in the sample }}$
- Here is a video specific to this topic. It includes examples and how to be most successful on the AP exam for the topic
7.08, 7.10, 7.11: Confidence Intervals and Proportions, Sample Size, and Margin of Error
https://sas.elluminate.com/site/external/jwsdetect/playback.jnlp?psid=2015-01-
24.1112.M.02B50E368656D296A2DCBFED1F5B9E.vcr\&sid=679

TESTS OF SIGNIFICANCE -- 1 and 2 proportion $Z$ test

| Actual AP Exam Expectations | Notes |
| :---: | :---: |
| 1. SHOW all steps of the test | 1. Parameter: same as confidence intervals for proportions <br> 2. Conditions ** For 2 proportion tests, you must check and state conditions for both populations** Same as confidence intervals for proportions <br> *SRS <br> *Normality <br> *Independence <br> Name the test: One proportion z test or 2 proportion z test <br> State the null ( Ho ) and alternative hypotheses ( Ha ) in context. <br> Note: $\mathrm{H}_{0}$ : null hypothesis is ALWAYS = to <br> $\mathrm{H}_{\mathrm{a}}$ : alternative is < > or $\neq$ <br> 3. Show all work: same as test <br> 4. Interpret the results in the context of the problem and make a connection to the given information. Remember the $\mathbf{3} \mathbf{C ' s}$ |
| 2. 1 proportion $z$ test | $\frac{\widehat{p}-p_{0}}{\sqrt{\frac{p_{0}\left(1-p_{0}\right)}{n}}}$ |


| 3. 2 proportion z test | $\begin{gathered} \frac{\hat{p}_{1}-\hat{p}_{2}}{\sqrt{\hat{p}(1-\hat{p}) \frac{1}{n_{1}}+\frac{1}{n_{2}}}} \\ * \hat{p}=\frac{\text { total } \# \text { of successes from both samples }}{\text { total number of individuals in both samples }} \end{gathered}$ |
| :---: | :---: |
| 6. State hypotheses in context | 1 proportion $z$ test: <br> Ho: The true population proportion of ___context_ is equal to $\qquad$ $\mathrm{p}=p_{0} *$ You must at least define what p is* <br> Ha: The true population proportion of $\qquad$ context is $<>\neq$ $\qquad$ $\mathrm{p}<>\neq p_{0}$ <br> 2 proportion z test: <br> Ho: The proportion of successes for context population 1 is equal to the proportion of successes for context population 2_. $p_{1}-p_{2}=0 \quad \text { or } \quad p_{1}=p$ <br> *You must define what $p_{1}$ and $p_{2}$ are* <br> Ha: The proportion of successes for context population 1 is $<>\neq$ to the proportion of successes for context population 2. $p_{1}-p_{2}<>\neq 0 \quad \text { or } \quad \overline{p_{1}<>\neq p_{2}}$ |
| 4. Interpret the results of the test | Conclusions should be given in terms of the context of the question. 1 proportion z test: Reject or Fail to reject the null hypothesis that proportion of context of the problem is $=$ to $p_{0}$ because the p value is $<>\neq$ level of significance. There is/is not sufficient evidence to suggest that $\qquad$ <br> 2 proportion $z$ test:: Reject or Fail to reject the null hypothesis that the difference in the proportion of context population 1 and context population 2 is equal because the $p$-value is $\rangle \neq$ level of significance. There is/is not sufficient evidence to suggest that restate the null hypothesis $\qquad$ |

- Errors: Type 1 error: rejecting Ho, when it is true

Type 2 error: failing to reject Ho, when it is false
If Ho is true, the probability of a Type 1 error $=\alpha$
The power of a test is the probability of correctly rejecting the Ho

- Here is a video specific to this topic. It includes examples and how to be most successful on the AP exam for the topic

Table entry for $p$ and $C$ is the point $t^{*}$ with probability $p$ lying above it and probability $C$ lying between $-t^{*}$ and $t^{*}$.


Table B $t$ distribution critical values

| df | Tail probability $p$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 25 | . 20 | . 15 | . 10 | . 05 | . 025 | . 02 | . 01 | . 005 | . 0025 | . 001 | . 0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | . 816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | . 765 | . 978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | . 741 | . 941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | . 727 | . 920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | . 718 | . 906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | . 711 | . 896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | . 706 | . 889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | . 703 | . 883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.69 C | 4.297 | 4.781 |
| 10 | . 700 | . 879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | . 697 | . 876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | . 695 | . 873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | . 694 | . 870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | . 692 | . 868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | . 691 | . 866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | . 690 | . 865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | . 689 | . 863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | . 688 | . 862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | . 688 | . 861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | . 687 | . 860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | . 686 | . 859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | . 686 | . 858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | . 685 | . 858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | . 685 | . 857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | . 684 | . 856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | . 684 | . 856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | . 684 | . 855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | . 683 | . 855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | . 683 | . 854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | . 683 | . 854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.03C | 3.385 | 3.646 |
| 40 | . 681 | . 851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | . 679 | . 849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | . 679 | . 848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | . 678 | . 846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | . 677 | . 845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | . 675 | . 842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $\infty$ | . 674 | . 841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
|  | 50\% | 60\% | 70\% | 80\% | 90\% | 95\% | 96\% | 98\% | 99\% | 99.5\% | 99.8\% | 99.9\% |

Confidence level $C$

